Anomaly-Free Gauged R-Symmetry

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Abstract

We review the gauging of an R-symmetry in local and global susy. We then construct the first anomaly-free models. We break the R-symmetry and susy at the Planck scale and discuss the low-energy effects. We include a solution to the mu-problem, and the prediction of observable effects at HERA. The models also nicely allow for GUT-scale baryogenesis and R-parity violation without the sphaleron interactions erasing the baryon-asymmetry.

1. Introduction

R-symmetries have been widely employed as discrete and global symmetries in susy. It is the purpose of this talk to discuss local anomaly-free R-symmetries. This paper is similar in spirit to [1] except we consider R-symmetries. It is based on the work [2]. For global susy theories the global R-transformations are [3]

$$\begin{array}{rcl} V_k(x,\theta,\bar\theta) & \rightarrow & V_k(x,\theta e^{-i\alpha},\bar\theta e^{i\alpha}), \\ S_i(x,\theta,\bar\theta) & \rightarrow & e^{in_i\alpha}S_i(x,\theta e^{-i\alpha},\bar\theta e^{i\alpha}), \end{array} \tag{1}$$

where V_k is a gauge vector multiplet S_i are left-handed chiral superfields. All gauginos transform non-trivially and with the *same* charge. The scalar fermions transform differently from their fermionic superpartners. The action for the superpotential $\int d^2\theta\, g(S_i)$, is invariant provided

$$g(S_i) \rightarrow e^{-2i\alpha} g(S_i). \tag{2}$$

It is *not* possible in global susy theories to promote the global R-invariance to a local one. (a) When the R-parameter α becomes local then

$$\theta \rightarrow \theta e^{-i\alpha(x)}, \quad \bar\theta \rightarrow \bar\theta e^{i\alpha(x)}, \tag{3}$$

which is a *local* superspace transformation. (b) For a local R-symmetry the R gauge vector boson V^R_μ couples to the R-gauginos λ^R

$$\mathcal{L} \sim \overline{\lambda}^R_L(\partial_\mu - ig_R V^R_\mu)\gamma^\mu\lambda^R_L + \overline{\lambda}^R_R(\partial_\mu + ig_R V^R_\mu)\gamma^\mu\lambda^R_R. \tag{4}$$

So $g_R\overline{\lambda}^R\gamma^\mu\gamma_5\lambda^R V^R_\mu$, must be in the Lagrangian but it isn't. In order to construct a susy Lagrangian containing this we must consider its susy transformation. It contains the term $g_R\epsilon^{\mu\nu\rho\sigma}\bar\epsilon\gamma_\mu\lambda^R F^R_{\nu\rho}V^R_\sigma = \epsilon^{\mu\nu\rho\sigma}\delta V^R_\mu V^R_\nu F^R_{\rho\sigma}$, since the susy variation of the gaugino term $\delta\lambda^R$ contains $\gamma^{\mu\nu}\epsilon F^R_{\mu\nu}$. This can not be cancelled without departing from global susy. (c) The R-symmetry generator R does not commute with the susy generator Q

$$[Q_\alpha, R] = i(\gamma_5)^\beta_\alpha Q_\beta. \tag{5}$$

The above equation can only hold for local R if the susy algebra is local.

We generalize the R-symmetry to the graviton multiplet as

$$e_\mu^m \to e_\mu^m, \quad \psi_\mu \to \exp(-i\alpha\gamma_5)\psi_\mu. \tag{6}$$

The R-gauge boson couples axially to the gravitino, the gauginos, and the chiral fermions. Such a Lagrangian was first constructed by Freedman [4]. The variation of (4) is now cancelled by

$$e^{-1}\mathcal{L} = \frac{i}{\sqrt{2}}\overline{\psi}_\rho\gamma^{\mu\nu}F^R_{\mu\nu}\gamma^\rho\lambda^R, \tag{7}$$

in the action since $\delta\psi_\mu$ contains $g_R V^R_\mu \gamma_5 \epsilon$. Ferrara *et al.* [5] showed that any R-invariant gauged action can be put into the canonical form of local susy with the function $\mathcal{G}(z_i, \bar{z}^i) = 3\ln(\frac{1}{3}\phi(z_i,\bar{z}^i)) - \ln|g(z_i)g^*(z^i)|$. The non-invariance of $\ln|g(z_i)g^*(z_i)|$ under R implies the appearance of the Fayet-Illiopoulos term in the D-term

$$\begin{aligned} g_R\mathcal{G}_{,}^{\,i} n_i z_i &= g_R(3\frac{\phi_{,}^{\,i}}{\phi} - \frac{g_{,}^{\,i}}{g})n_i z_i. &(8)\\ n_i z_i g_{,}^{\,i} &= 3\xi g. &(9)\end{aligned}$$

This leads to a cosmological constant of order κ^4 which fixes the scale of $U(1)_R$ -breaking.

3. Conditions for the Cancellation of Anomalies

3.1 Family Independent Gauged R-symmetry

We construct a anomaly-free $N=1$ local susy theory with the gauge group $G_{SM}\times U(1)_R \equiv SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_R$. The matter chiral multiplets are

$$\begin{aligned} &L: \quad (1,2,-\frac{1}{2},l), \quad \bar{E}:(1,1,1,e), \quad Q:(3,2,\frac{1}{6},q), \quad \bar{U}:\bar{3},1,-\frac{2}{3},u), &(10)\\ &\bar{D}: \quad (\bar{3},1,\frac{1}{3},d), \quad H:(1,2,-\frac{1}{2},h), \quad \bar{H}:(1,\bar{2},\frac{1}{2},\bar{h}), \quad N:(1,1,0,n), \quad z_m:(1,1,0,z_m),\end{aligned}$$

where we have indicated the gauge quantum numbers. The $U(1)_R$ quantum numbers are for the chiral fermions. The superpotential in the observable sector has the form

$$g^{(O)} = h_E^{ij}L_i\bar{E}_jH + h_D^{ij}Q_i\bar{D}_jH + h_U^{ij}Q_i\bar{U}_j\bar{H} + h_N NH\bar{H}, \tag{11}$$

where h_E, h_D, h_U, h_N are the Yukawa couplings. We assume the theory conserves R-parity. The requirement that comes from R-invariance for $g^{(O)}$ is

$$l+e+h = -1, \quad q+d+h=-1, \quad q+u+\bar{h}=-1, \quad n+h+\bar{h}=-1. \tag{12}$$

The equations for the absence of the $U(1)_Y - U(1)_R$ anomalies give

$$\begin{aligned} C_1 \equiv 3[\frac{1}{2}l+e+\frac{1}{6}q+\frac{4}{3}u+\frac{1}{3}d]+\frac{1}{2}(h+\bar{h}) &= 0, &(13)\\ 3[-l^2+e^2+q^2-2u^2+d^2]-h^2+\bar{h}^2 &= 0, &(14)\\ 3[2l^3+e^3+6q^3+3u^3+3d^3]+2h^3+2\bar{h}^3+16+n^3+\sum z_m^3 &= 0. &(15)\end{aligned}$$

The term $16=13+3$ is due to the 13 gauginos as well as the gravitino. The absence of the mixed $U(1)_R\,SU(2)_L$ and $U(1)_R - SU(3)_C$ anomalies implies

$$C_2 \equiv 3[\frac{1}{2}l+\frac{3}{2}q]+\frac{1}{2}(h+\bar{h})+2=0. \tag{16}$$

$$C_3 \equiv 3[q+\frac{1}{2}u+\frac{1}{2}d]+3=0. \tag{17}$$

The cancellation of the mixed gravitational anomaly [illegible] requires [illegible],

$$3[2l + e + 6q + 3u + 3d] + 2(h + \bar{h}) - 8 + n + \sum z_m = 0. \tag{18}$$

The term $-8 = 13 - 21$ is due to the 13 gauginos as well as the gravitino. These ten equations do not have a solution, independently of the singlet charges.

3.2 Green-Schwarz Anomaly Cancellation

The Green-Schwarz mechanism of anomaly cancellation relies on coupling the system to a linear multiplet $(B_{\mu\nu}, \phi, \chi)$ where $B_{\mu\nu}$ is an antisymmetric tensor [7]. The non-invariant part of the gauge transformations of the action of $B_{\mu\nu}$ are of exactly the same form as the mixed gauge anomalies C_1, C_2, and C_3. The combined action is gauge invariant provided $C_1/k_1 = C_2/k_2 = C_3/k_3$. The k_i are the Kač-Moody levels of the gauge algebra. For $k_2 = k_3$ the anomaly cancellation conditions are compatible if $C_2 = C_1 + 6$. We can simplify the equations by assuming that $C_2/C_1 = 3/5$, $(\sin^2\theta_w = \frac{3}{8})$. Then $C_1 = -15$, $C_2 = C_3 = -9$. The anomaly cancellation equations can all be expressed in terms of one variable $l' = \frac{30}{7}\cdot l$ beyond the quantum numbers of the singlet fields z_m. The remaining equations (including the linear multiplet) are

$$-80 + \frac{3}{2}l' + \sum z_m = 0, \quad -\frac{8004}{9} - 24l' + \frac{19}{5}l'^2 + \frac{3}{8}l'^3 + \sum z_m^3 = 0, \tag{19}$$

There is no rational solution for zero or one singlet. We have performed a numerical scan for three singlets and found no solution. We conclude that it is not possible to cancel the anomaly via the Green-Schwarz mechanism with a small number of singlets.

3.3 Non-Singlet Field Extensions

We allow for extra generations N_g and pairs of Higgs doublets N_h. The anomaly equations are

$$\begin{array}{llllll} h = -(l+e+1), & \bar{h} = l+e-1, & q = -\frac{2}{9} - \frac{1}{3}l, & \\ d = \frac{2}{9} + \frac{4}{3}l + e, & u = \frac{2}{9} - \frac{2}{3}l - e, & n = 1, & o_c = -1. \end{array} \tag{20}$$

$$3(2l+e) - 19 + \sum_i z_i = 0, \quad 3(2l+e)^3 + 13 + \sum_i z_i^3 = 0. \tag{21}$$

We found many solutions with four singlets, *e.g.* $(2l+e, z_1, z_2, z_3, z_4) = (1, -\frac{47}{3}, -\frac{25}{3}, 3, 13)$. The fermionic component of the octet chiral superfield has R-charge -1 and the scalar potential of the octet is unconstrained and typically breaks $SU(3)_c$.

3.4 Family Dependent Gauged $U(1)_R$ Symmetry

We denote the R-quantum number of the matter fields by e_i, l_i, q_i, u_i, and d_i, $i = 1,2,3$. We assume a left-right symmetry

$$e_i = l_i, \quad u_i = d_i = q_i, \quad i = 1,2,3. \tag{22}$$

We assume that only the fields of the third generation enter the superpotential.

$$g^{(O)} = h_E^{33} L_3 \bar{E}_3 H + h_D^{33} Q_3 \bar{D}_3 H + h_U^{33} Q_3 \bar{U}_3 \bar{H} + h_N N H \bar{H}. \tag{23}$$

The masses for the first and second generation will be generated after the breaking of some symmetry, possibly the R-symmetry. The anomaly equations are solved in the visible fields and reduce to

$$\begin{array}{lll} h = \bar{h} = -1, & q_3 = l_3 = 0, & \\ l_2 = \frac{5}{2} - l_1, & q_2 = -(\frac{3}{2} + q_1), & n = 1. \end{array} \tag{24}$$

$$\frac{45}{2}l_1(l_1 - \frac{5}{2}) - 54q_1(q_1 + \frac{3}{2}) + \frac{155}{8} + \sum z_m^3 = 0, \quad \sum z_m = \frac{43}{2}. \tag{25}$$

unacceptable cosmological constant. Some of the fermionic charges of the observable fields are < -1. The potential then requires fine-tuning to guarantee weak-scale sfermion masses. For two singlets we find many solutions. These solutions have negative singlet charges but q_1 or $q_2 < -1$. We found one three singlet solution.

$$\{(q_1,q_2,q_3);(l_1,l_2,l_3);(z_1,z_2,z_3)\} = \left\{(-1,-\frac{1}{2},0);(\frac{1}{2},2,0);(-\frac{115}{3},26,\frac{203}{6})\right\}. \tag{26}$$

There are three further solutions obtained by $q_1 \leftrightarrow q_2$ and $l_1 \leftrightarrow l_2$. For four singlets we find many solutions. The solutions with observable field fermionic charges greater than -1 are

$$q_1 = -1, \quad l_1 = \frac{n}{6}, \quad n=-6,...,6,\, n\neq 0 \tag{27}$$

$$q_1 = -\frac{5}{6}, \quad l_1 = \frac{n}{6}, \quad n=-6,...,6,\, n\neq -4,0,4 \tag{28}$$

The other charges are given in [2]. The solutions with $q_1=-1$ has an unacceptable level of proton decay.

4. Susy and R-symmetry Breaking

To have a realistic model both susy and R-symmetry must be broken at low energies. A Fayet-Illiopoulos term is necessarily present in the D-term[1] and we have a cosmological constant of the order of the Planck scale. In a realistic model to lowest order the condition

$$< n_i z^i z_i > + \frac{4}{\kappa^2} = 0, \tag{29}$$

must be satisfied [8]. At least one chiral superfield must have negative R-charge. Only the singlets should get a vev at the Planck scale. The most general polynomial with R-charge 2 for the three singlet solution is given by

$$g'(z_1,z_2,z_3) = \frac{1}{\kappa^3}\left(a_1(\kappa z_1)^{10}(\kappa z_2)(\kappa z_3)^{10} + a_2(\kappa z_1)^{25}(\kappa z_2)^{14}(\kappa z_3)^{16}\right. \tag{30}$$

$$\left. + a_3(\kappa z_1)^{33}(\kappa z_2)^{7}(\kappa z_3)^{30} + a_4(\kappa z_1)^{41}(\kappa z_3)^{44} + ...\right). \tag{31}$$

We take the arbitrary parameters $a_k = \mathcal{O}(1)$. We can not break susy via the Polonyi mechanism since a constant is not R-invariant. We need at least three non-zero parameters a_k in g' then it is possible to find solutions for which the total potential V is positive semi-definite with the value zero at the minimum, and where the D-term is also zero at the minimum. The R-gauge vector boson mass is then of order the Planck mass. The total superpotential and potential are

$$g = g'(z_1,z_2,z_3) + g^{(O)}(S_i), \tag{32}$$

$$V = \frac{1}{\kappa^4} e^{\mathcal{G}}\left(\mathcal{G}_{,}^{-1a}{}_{b}\mathcal{G}_{,a}\mathcal{G}_{,}^{b} - 3\right) + \frac{1}{2}\tilde{g}^2 \mathcal{R}e f^{-1}_{\alpha\beta}\left(\mathcal{G}_{,}^{a}(T^\alpha z)_a\right)\left(\mathcal{G}_{,}^{b}(T^\beta z)_b\right). \tag{33}$$

For the three-singlet model we thus obtain the D-term as

$$g_R^2 \frac{1}{8}\left(\frac{2}{3}\right)^2 \left(-\frac{112}{3}|z_1|^2 + 27|z_2|^2 + \frac{209}{6}|z_3|^2 + \frac{4}{\kappa^2}\right)^2 \tag{34}$$

In g' it is clear that there is no symmetry in z_1, z_2, z_3 and their vevs will be unequal. For the D-term to vanish at the minimum we must have $|z_2| < \frac{112}{81}|z_1|$, and $|z_3| < \frac{224}{209}|z_1|$. By fine-tuning

[1] Here we have assumed that the kinetic energy is minimal and of the form $y = \frac{\kappa^2}{2} z_i z^i + ...$

Then if we start with the natural Planck scale $\frac{1}{\kappa}$, the effective value of g' will be $\frac{m_s}{\kappa^2}$, where $m_s = \frac{1}{\kappa^2}\left(\frac{1}{5}\right)^{21}\left(\frac{1}{2}\right)^{11}$ is of order $\mathcal{O}(10^2\, GeV)$. We shall assume that $z_1, z_2, z_3 \approx \mathcal{O}(\frac{1}{\kappa})$ with coefficients less than one, so that when these fields are integrated out one gets $<\kappa^2 g'> = m_s$.

By integrating the hidden sector fields z_1, z_2, z_3 one obtains the effective potential as a function of the light fields z_i. It was shown in [9] that the low-energy effective potential is identical to that of the MSSM

$$\begin{aligned} V &= |\hat{g}_{,i}|^2 + m_s^2|z_i|^2 + m_s\left(z_i\hat{g}_{,i} + (A-3)\hat{g} + h.c.\right) \\ &\quad + \frac{1}{8}g^2\left(H^*\sigma^a H + \bar{H}^*\sigma^a\bar{H}\right)^2 + \frac{1}{8}g'^2\left(H^*H - \bar{H}^*\bar{H}\right)^2 \end{aligned} \tag{35}$$

The three singlet solution is problematic with the $\bar{U}\bar{D}\bar{D}$ couplings as will be clear in the next section. Therefore we must consider the four singlet solutions which we required to avoid such a problem. The superpotentials for the ten different classes are given in Table 2.

As before we have to tune the parameters a_k so that the potential is positive definite and so that $|z_1|, ..., |z_4| \approx \mathcal{O}(\frac{1}{\kappa})$ with coefficients less than one so as to induce a scale such that $<\kappa^2 g'> = m_s = \mathcal{O}(10^2\, GeV)$. The effective potential takes the same form as in the three singlet case, but with different R-numbers for the squarks and sleptons.

5. Applications to R-parity Violation

When extending the Standard Model to susy new dimension four Yukawa couplings are allowed which violate baryon- and lepton-number.

$$L_iL_j\bar{E}_k, \quad L_iQ_j\bar{D}_k, \quad \bar{U}_i\bar{D}_j\bar{D}_k, \quad \tilde{\mu}L_i\bar{H}, \tag{36}$$

where $\tilde{\mu}$ is a dimensionful parameter. The indices i, j, k are generation indices. For the three singlet solution we obtain the following additional terms

$$LL\bar{E}: \quad \text{none}; \quad \bar{U}\bar{D}\bar{D}: \ \bar{U}_3\bar{D}_1\bar{D}_3,\ \bar{U}_2\bar{D}_2\bar{D}_3, \tag{37}$$

$$LQ\bar{D}: \quad L_1Q_1\bar{D}_2,\ L_1Q_2\bar{D}_1;\ L_3Q_1\bar{D}_3,\ L_3Q_3\bar{D}_1,\ L_3Q_2\bar{D}_2, \tag{38}$$

$LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ terms together lead to a dangerous level of proton decay. We thus exclude the three singlet solution. Similarly we also exclude the four singlet solutions with $q_1 = -1$. For the ten models of Table 1 [2] we find the following sets of gauge invariant R-parity violating dimension-four terms

$$\begin{aligned} I: &\quad L_1L_3\bar{E}_3,\ L_1Q_3\bar{D}_3 \quad III: L_1L_3\bar{E}_1 \quad IV: L_1Q_2\bar{D}_3,\ L_1Q_3\bar{D}_2, \\ V: &\quad L_1Q_1\bar{D}_3,\ L_1Q_3\bar{D}_1, \quad VII: L_1Q_2\bar{D}_2, \quad VIII: L_1Q_1\bar{D}_2,\ L_1Q_2\bar{D}_1, \quad X: L_1\bar{H}. \end{aligned} \tag{39}$$

We have models with only $LL\bar{E}$ type couplings, others with only $L_i\bar{H}$ or $LQ\bar{D}$ couplings. We also have three sets II, VI, IX where R-parity is conserved. Thus there is no logical connection between a conserved R-symmetry and the status of R-parity.

The $L_{1,2}\bar{H}$ term has a dimensionful coupling $\tilde{\mu}$ similar to the μ term of the MSSM. In order to avoid a further hierarchy problem we require the absence of $L_i\bar{H}$ terms and therefore exclude the models X, X'.

Interestingly enough, most of the models predict sizeable $L_{1,2}Q_i\bar{D}_j$ interactions. The first set leads to resonant squark production at HERA which has been investigated in detail in [10]. This should be observable with an integrated luminosity of about $100\, pb^{-1}$ for squark masses below $275\, GeV$. The second set also lead to observable signals at HERA even for very small couplings as discussed in [11]. These models should also be observable at a hadron collider [12].

We point out that only in model I we have additional terms L_1HN. These conserve R-parity provided N is interpreted as a right-handed neutrino. L_1HN is a Dirac neutrino mass and requires a very small Yukawa coupling. We thus exclude model I. It is interesting to note that eventhough for the Higgs Yukawa couplings the third generation is dominant this is not necessarily the case for the R_p violating interactions.

It is worth pointing out that models **I, III, IV, V, VII, VIII** are just of the type postulated in [13]. In order to maintain GUT-scale baryogenesis at low-energies despite the sphaleron interactions and have R-parity violation at a measurable level at least one lepton number had to be conserved. This is guaranteed by an anomaly-free gauge symmetry in our models.

Finally we point out that the present work can easily be extended to include a solution to the mu problem [14]. We must drop the N-field. Then the R-charge of H_1H_2 is just 0, so it is disallowed in the superpotential but allowed in the Kähler potential. The corresponding equations have solutions for 4 extra singlets. H_1H_2 do not couple to Planck scale fields.

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